Statistical analysis of the exchange rate of Bitcoin

Stephen Chan

Department of Mathematics and Statistics, American University of Sharjah, Sharjah

January 5, 2018

Chan, S., Chu, J and Nadarajah, S. (2015). Statistical analysis of the exchange rate of Bitcoin. *PloS one*, 10 (7)

Outline

- Some cryptocurrencies;
- What is Bitcoin;
- Data;
- Distributed fitted;
- Results;
- Prediction;
- References

Some cryptocurrencies

- Market capitalization value is more than $500 Billion.
- Over 100 different cryptocurrencies in the market.
- Bitcoin (45 percent);
- Ripples (20 percent);
- Ethereum (16 percent).
- Litecoin (2 percent);
- Dash (1.8 percent);

What is Bitcoin?

- Bitcoin is a form of cryptocurrency;
- Introduced in 2009 by Satoshi Nakamoto (there is controversy though);
- Highly volatile; (2009 = \$0.5, 2017 = \$20000)

Some images of Bitcoin

Figure: ATM in California.

Some images of Bitcoin

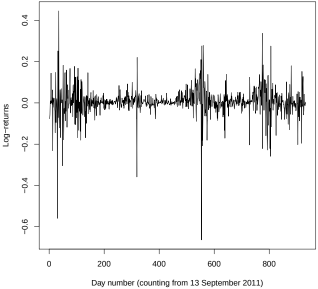
Figure: Cafe in Delft, Netherlands.

Some images of Bitcoin

Figure: Subway in Allentown, PA.

Data

- Motivation and contribution for analysing of Bitcoin
- Data: Bitstamp exchange from Sept 2011- May 2014.



Descriptive Statistic of Data

Statistics	Bitcoin	OZ	BR	CA	CH	EU	UK	JP
Minimum	−0.664	-0.067	-0.118	-0.050	-0.055	-0.046	-0.045	-0.046
First quartile	−0.012	-0.004	-0.005	-0.003	-0.004	-0.004	-0.003	-0.003
Median	0.004	-0.0002	-0.00005	0	0	0	-0.00006	0.00009
Mean	0.005	-0.00005	0.0001	-0.00005	-0.0001	-0.00004	0.00001	0.00004
Third quartile	0.025	0.004	0.004	0.003	0.004	0.003	0.003	0.004
Maximum	0.446	0.088	0.097	0.043	0.085	0.038	0.039	0.037
Interquartile range	0.037	0.008	0.009	0.006	0.008	0.007	0.006	0.007
Range	1.109	0.155	0.215	0.094	0.139	0.085	0.084	0.083
Skewness	−1.503	0.866	0.110	-0.076	0.358	-0.145	0.055	-0.253
Kurtosis	22.425	12.707	13.826	5.765	9.170	2.662	4.413	3.889
Standard deviation	0.069	0.008	0.010	0.006	0.007	0.006	0.006	0.006
Variance	0.005	0.00007	0.0001	0.00003	0.00005	0.00004	0.00003	0.00004
Coefficient of variation	15.156	-157.291	96.649	-108.894	-57.625	-143.498	419.938	147.998

Table: Summary statistics log-returns of the exchange rate of Bitcoin versus those of Australian Dollar, Brazilian Real, Canadian Dollar, Swiss Franc, Euro, British Pound and Japanese Yen.

Some of the distributions fitted

Studies from Coppes (1995) and Corlu *et al. (2015)*

- Heavy tailed: Generalised t, Generalised hyperbolic and Normal inverse Gamma.
- Light tailed: Normal, Exponential power , Laplace and Logistic.
- Several of these distributions are nested.

Distributions fitted: Normal

The normal distribution with

$$f(x) = \frac{1}{\sqrt{2\pi}\sigma} \exp\left\{-\frac{(x-\mu)^2}{2\sigma^2}\right\}$$

for $-\infty < x < \infty$, $-\infty < \mu < \infty$ and $\sigma > 0$.

Distributions fitted: Asymmetric exponential power

The asymmetric exponential power distribution with

$$f(x) = C \begin{cases} \exp\left\{-\frac{1}{p_1}\left[\frac{\mu - x}{2\sigma\alpha}\right]^{p_1}\right\}, & \text{if } x \le \mu, \\ \exp\left\{-\frac{1}{p_2}\left[\frac{x - \mu}{2\sigma(1-\alpha)}\right]^{p_2}\right\}, & \text{if } x > \mu \end{cases}$$

for $-\infty < x < \infty$, $-\infty < \mu < \infty$, $\sigma > 0$, $\alpha > 0$, $p_1 > 0$ and $p_2 > 0$, where C is given by

$$C = \frac{1}{2\sigma\alpha A_0\left(p_1\right) + 2\sigma(1-\alpha)A_0\left(p_2\right)}.$$

Distributions fitted: Asymmetric Student's t

The asymmetric Student's t distribution with

$$f(x) = \frac{1}{\sigma} \begin{cases} \frac{\alpha}{\alpha^*} K(\nu_1) \left\{ 1 + \frac{1}{\nu_1} \left[\frac{x-\mu}{2\sigma\alpha^*} \right]^2 \right\}^{-\frac{\nu_1+1}{2}}, & \text{if } x \leq \mu, \\ \frac{1-\alpha}{1-\alpha^*} K(\nu_2) \left\{ 1 + \frac{1}{\nu_2} \left[\frac{x-\mu}{2\sigma(1-\alpha^*)} \right]^2 \right\}^{-\frac{\nu_2+1}{2}}, & \text{if } x > \mu \end{cases}$$

for $-\infty < x < \infty$, $-\infty < \mu < \infty$, $0 < \alpha < 1$, $\nu_1 > 0$ and $\nu_2 > 0$, where

$$\alpha^* = \frac{\alpha K(\nu_1)}{\alpha K(\nu_1) + (1-\alpha) K(\nu_2)}.$$

Distributions fitted: Generalized hyperbolic

The generalized hyperbolic distribution with

$$f(x) = \frac{(\gamma/\delta)^{\lambda}\,\alpha^{1/2-\lambda}}{\sqrt{2\pi}K_{\lambda}(\delta\gamma)}\left[\delta^2 + (x-\mu)^2\right]^{\lambda-1/2} K_{\lambda-1/2}\left(\alpha\sqrt{\delta^2 + (x-\mu)^2}\right)$$

for $-\infty < x < \infty$, $-\infty < \mu < \infty$, $-\infty < \lambda < \infty$, $\delta > 0$, $\alpha > 0$ and $\beta > 0$, where $\gamma = \sqrt{\alpha^2 - \beta^2}$.

Statistical analysis of the exchange rate of Bitcoin

Methodology:

- Method of maximum likelihood
- -Log likelihood, AIC, BIC, CAIC, KS and Likelihood ratio test.

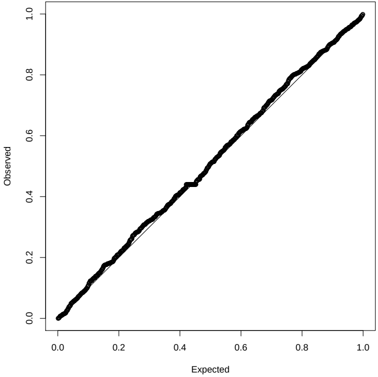
Distribution	$-\ln L$	AIC
Normal	1196.425	2396.851
Skew normal	-1196.425	-2386.851
Skew t	-1556.337	-3104.674
Generalized t	-1565.963	-3123.926
SEP	-1560.14	-3112.28
AEP	-1567.824	-3125.648
AST	-1558.258	-3106.516
NIG	-1565.278	-3122.557
Generalized hyperbolic	-1570.229	-3130.458

Results and discussions:

- Best model was the Generalized hyperbolic.
- The parameters estimates and standard errors are:
  $\widehat{\mu} = 2.948 \times 10^{-3} \left(8.964 \times 10^{-4}\right)$,
  $\widehat{\delta} = 1.217 \times 10^{-2} \left(2.578 \times 10^{-3}\right)$, $\widehat{\alpha} = 7.731(1.517)$,
  $\widehat{\beta} = 3.447 \times 10^{-1} \left(5.186 \times 10^{-1}\right)$,
  $\widehat{\lambda} = -1.390 \times 10^{-1} \left(1.112 \times 10^{-1}\right)$

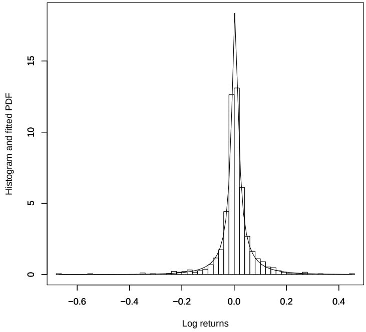
Statistical analysis of the exchange rate of Bitcoin

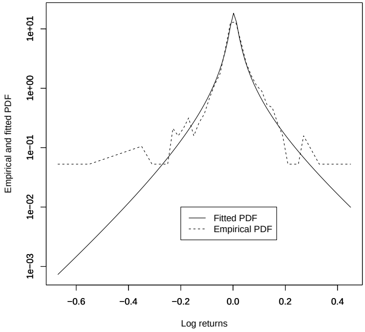
- The probability plot of the fitted generalized hyperbolic distribution



Statistical analysis of the exchange rate of Bitcoin

- Empirical histogram and fitted pdf of the generalised hyperbolic distribution (left), Empirical pdf and fitted pdf of the generalized hyperbolic distribution plotted on log scale (right).





Statistical analysis of the exchange rate of Bitcoin

- The VaR and the ES with probability p can be estimated by:

$$\frac{\left(\widehat{\gamma}/\widehat{\delta}\right)^{\widehat{\lambda}}\widehat{\alpha}^{1/2-\widehat{\lambda}}}{\sqrt{2\pi}K_{\widehat{\lambda}}\left(\widehat{\delta}\widehat{\gamma}\right)}\int_{-\infty}^{\widehat{\mathrm{VaR}}_p}\left[\widehat{\delta}^2+(x-\widehat{\mu})^2\right]^{\widehat{\lambda}-1/2}K_{\widehat{\lambda}-1/2}\left(\widehat{\alpha}\sqrt{\widehat{\delta}^2+(x-\widehat{\mu})^2}\right)dx=p$$

and

$$\widehat{\mathrm{ES}}_p=\frac{1}{p}\int_0^p\widehat{\mathrm{VaR}}_q dq,$$

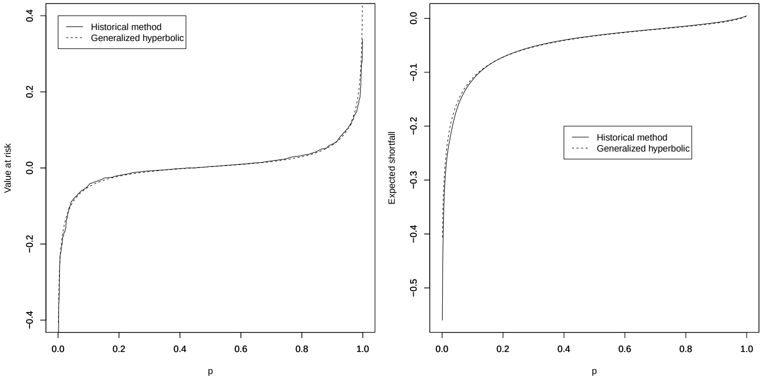
VaR and ES

p	VaR	ES
0.1	-5.015×10^{-2}	-1.138×10^{-1}
0.01	-2.043×10^{-1}	-2.926×10^{-1}
0.001	-4.108×10^{-1}	-5.105×10^{-1}
0.0001	-6.420×10^{-1}	-7.477×10^{-1}
0.00001	-8.865×10^{-1}	-9.960×10^{-1}
0.9	6.023×10^{-2}	-9.360×10^{-3}
0.99	2.282×10^{-1}	3.432×10^{-3}
0.999	4.539×10^{-1}	3.975×10^{-3}
0.9999	7.065×10^{-1}	4.446×10^{-3}
0.99999	9.739×10^{-1}	4.525×10^{-3}

Table: Fitted estimates of VaR and ES.

Statistical analysis of the exchange rate of Bitcoin

The plot of $\widehat{\mathrm{VaR}}_p$ versus p (left) and the plot of $\widehat{\mathrm{ES}}_p$ versus p (right)



Statistical analysis of the exchange rate of Bitcoin

Distribution	$\alpha = 0.001$	$\alpha = 0.999$
Normal	5.908	5.335
Student t	3.253	3.599
Generalized t	0.589	1.686
SEP	1.739	2.624
AEP	0.093	1.492
SST	1.899	2.845
AST	1.884	2.786
NIG	0.925	2.236
Hyperbolic	4.715	4.208
Generalized hyperbolic	0.052	0.134

Table: The backtest measure for the fitted distributions.

Statistical analysis of the exchange rate of Bitcoin

Predictions: Let Y_i denote the exchange rate on the ith day counting from the 13th of September 2011. Then $X_i = \ln Y_i - \ln Y_{i-1}$ is the log-return on the ith day. So

$$Y_n = 5.97 + \exp\left(\sum_{i=1}^{n} X_i\right) = 5.97 + \exp(T)$$

the cdf of T can be expressed as

$$F_T(t) = \frac{1}{2} - \frac{\widehat{\gamma}^{n\widehat{\lambda}}}{\pi K_{\widehat{\lambda}}^{n}\left(\widehat{\delta}\widehat{\gamma}\right)} \int_0^{\infty} s^{-1} \mathrm{Im}\left\{ \frac{K_{\widehat{\lambda}}^{n}\left(\widehat{\delta}\sqrt{\widehat{\alpha}^2 - \left(\widehat{\beta} + \mathrm{i}s\right)^2}\right) \exp\left[\mathrm{i}s(n\mu - t)\right]}{\left[\widehat{\alpha}^2 - \left(\widehat{\beta} + \mathrm{i}s\right)^2\right]^{-n\widehat{\lambda}/2}} \right\} ds,$$

where $\mathrm{i} = \sqrt{-1}$ and $\mathrm{Im}(\cdot)$ denotes the imaginary part. The cdf of Y_n is therefore

$$F_{Y_n}(y) = F_T\left(\ln(y - 5.97)\right).$$

Statistical analysis of the exchange rate of Bitcoin

- Predictions for the exchange rate of Bitcoin at day n
- n=1000(approximately three years).
- The exchange rate in about six years from the 13th of September 2011 could exceed 10172920 with 1 percent chance and could be less than 13.360 with 1 percent chance.

Percentile level	$n = 1000$	$n = 2000$	$n = 3000$	$n = 4000$	$n = 5000$
0.1	11.904	182.635	6863.276	305360	14844735
0.01	6.599	13.360	146.519	3435.949	98169.76
0.001	6.092	6.696	14.163	134.777	2508.388
0.0001	6.002	6.077	6.760	14.613	128.040
0.00001	5.980	5.990	6.074	6.798	14.838
0.9	1467.006	425537.8	95054327	18508171925	3.301×10^{12}
0.99	13791.29	10172920	4637660718	1.648×10^{12}	4.992×10^{14}
0.999	71142.51	103590788	79559056488	4.388×10^{13}	1.958×10^{16}
0.9999	274621.4	699752429	8.256×10^{11}	6.539×10^{14}	4.014×10^{17}
0.99999	887173.1	3674309279	6.293×10^{12}	6.824×10^{15}	5.526×10^{18}

Applications for Financial Institutions

Importance of results

- Cryptocurrencies such as Bitcoin and other cryptocurrency and many others exhibit extreme volatility especially when we look at their inter daily price. This kind of investment is suited for risk seeking investors who would like to incorporate such investment in their long term portfolio or who are looking for an alternative way to invest or enter in the technology market.
- Some financial Institutions:
  Skandiabanken is the largest internet-based bank in Scandinavia.
  Falcon Private Bank, a multi - million pound asset management - Zurich.
  Saxo Bank, Denmark-based multi - million asset and trading institution.

Statistical analysis of the exchange rate of Bitcoin

Conclusions:

- The generalized hyperbolic distribution gives the best fit.
- Predictions for the log-returns of the exchange rate based on the VaR and the ES.
- Predictions for the exchange rate at future times.
- The results are important for investment and risk management purposes.

Thank You!